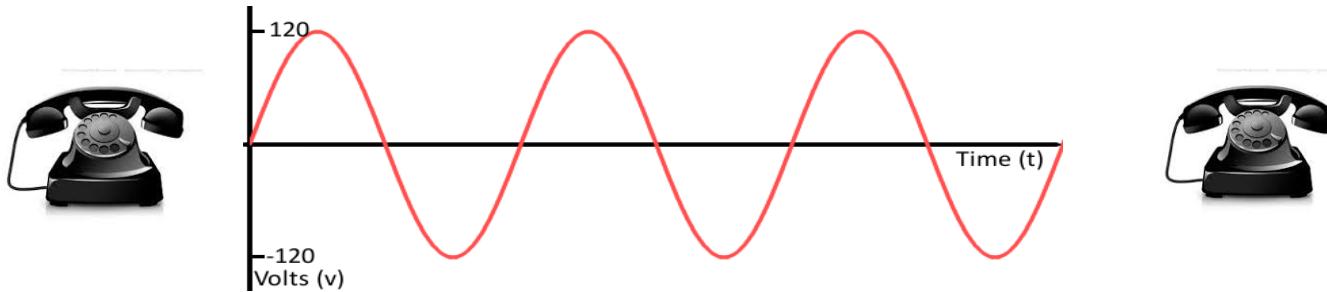


Bits, Bytes, Ints

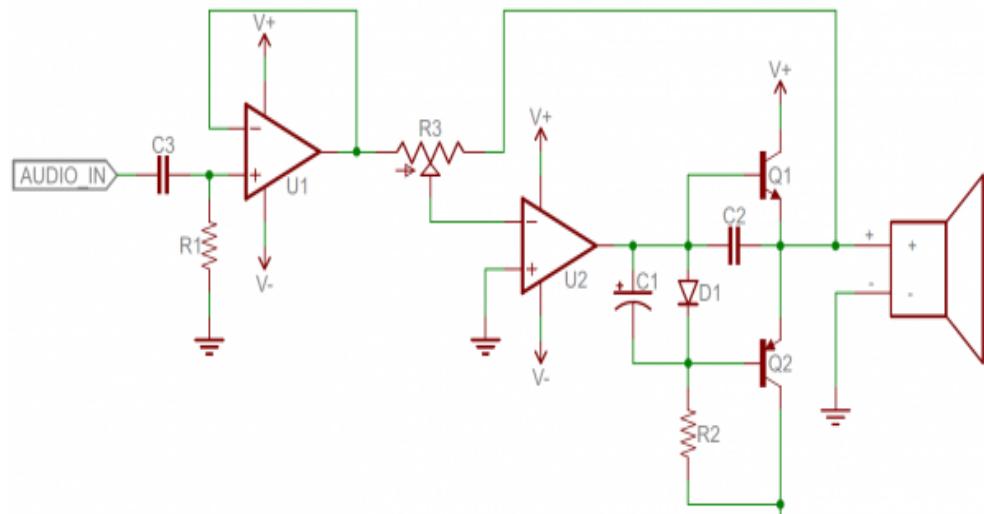
Jinyang Li

Slides are based on Tiger Wang's class

The world has moved away from analog signal to ...



Analog signals: smooth and continuous

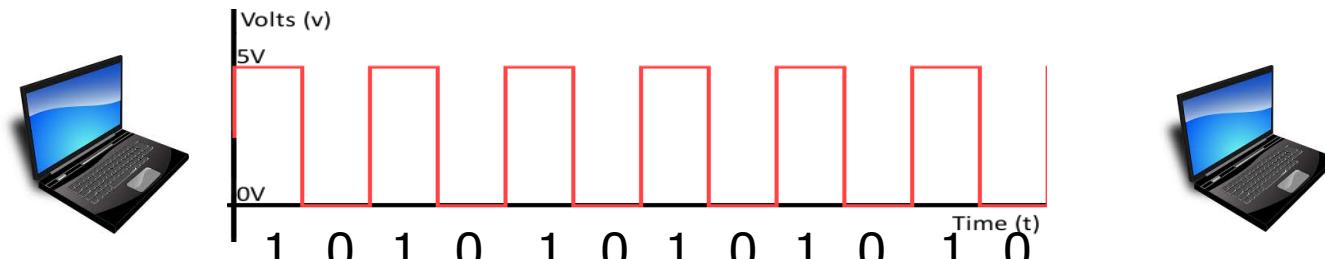


Problems

1. Difficult to design
2. Susceptible to noise

Analog components: resistors, capacitors, inductors, diodes, e.t.c...

... to digital



Digital signals: discrete (encode sequence of 0s and 1s)

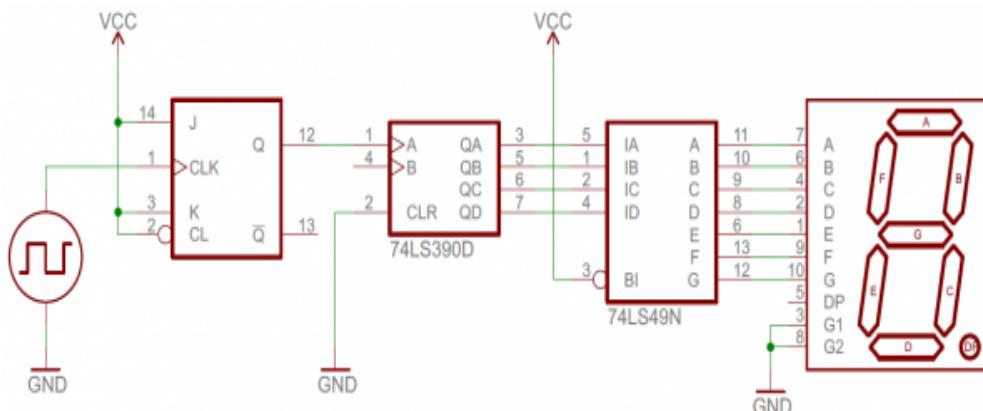
Advantages

1. Easier to design

- Simple
- Integrate millions on a single chip

2. Reliable

- Robust to noise



Digital components: transistors, logic gates ...

Using bits to represent everything

Bit = Binary digit, 0 or 1

A bit is too small to be used much

- A bit has two values; the English alphabet has 26 value (characters)

Using bits instead of bit

- Group bits together
- different possible bit patterns represent different “values”

Question

- How many values can a group of 2 bits represent?

0	0
0	1
1	0
1	1

4

- How many values can a group of n bits represent?

2^n

Allow us to represent
 $0, 1, 2, \dots (2^n - 1)$

Represent non-negative integer

bits: $b_{n-1}b_{n-2}\dots b_2b_1b_0$

Question: how to map each bit pattern to a unique integer in $[0, 2^n - 1]$?

Solution: Base-2 representation

$$b_{n-1}b_{n-2}\dots b_2b_1b_0 = \sum_{i=0}^{n-1} b_i * 2^i$$

b_i is bit at i-th position
(from right to left,
starting $i=0$)

Most significant bit (MSB)

The bit position has the greatest value

Bits 01010

MSB ?

Bits 11011010

MSB ?

Most significant bit (MSB)

The bit position has the greatest value
– The leftmost bit

Bits 01010

MSB 0

Bits 11011010

MSB 1

Least significant bit (LSB)

The bit position has the least value
– The rightmost bit

Bits 01010

MSB 0

Bits 11011010

MSB 0

Examples

Bits 0110

Value $0*2^3 + 1*2^2 + 1*2^1 + 0*2^0 = 6$

Bits 1110

Value ?

$$1*2^3+1*2^2+1*2^1+0*2^0 = 14$$

Byte

Each memory unit has multiple bits

- Dr. Werner Buchholz in July 1956
- Byte sizes from 1 bit to 48 bits have been used in the history

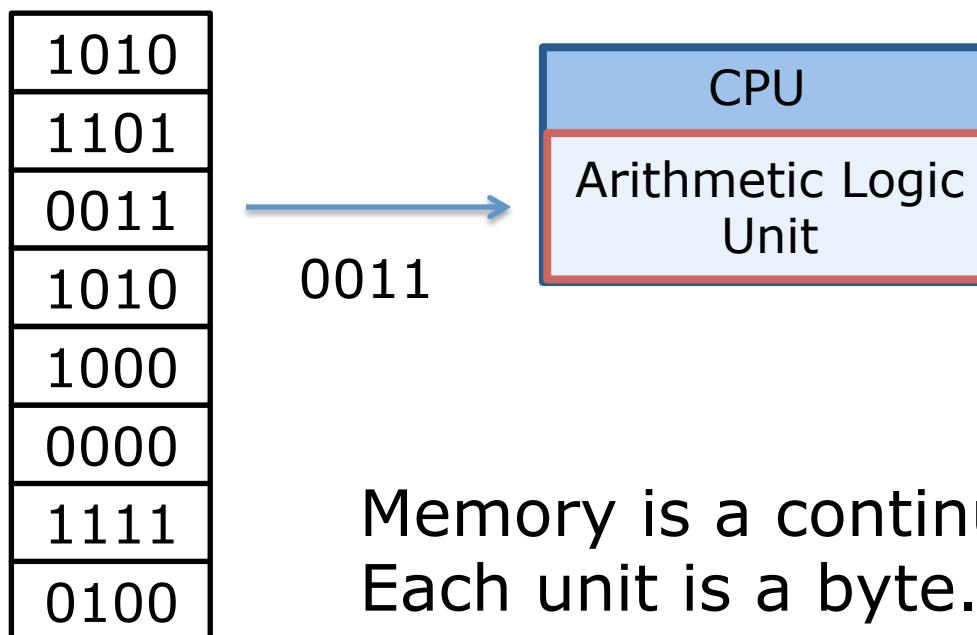


Byte

Each memory unit has multiple bits



- Dr. Werner Buchholz in July 1956
- Byte sizes from 1 bit to 48 bits have been used in the history



Memory is a continuous array.
Each unit is a byte.

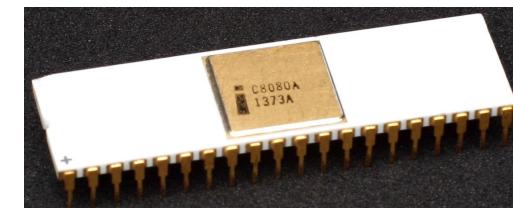
Memory

Byte – 8 bits chunk



IBM System/360, 1964

Introduce



Intel 8080, 1974

Widely adopted

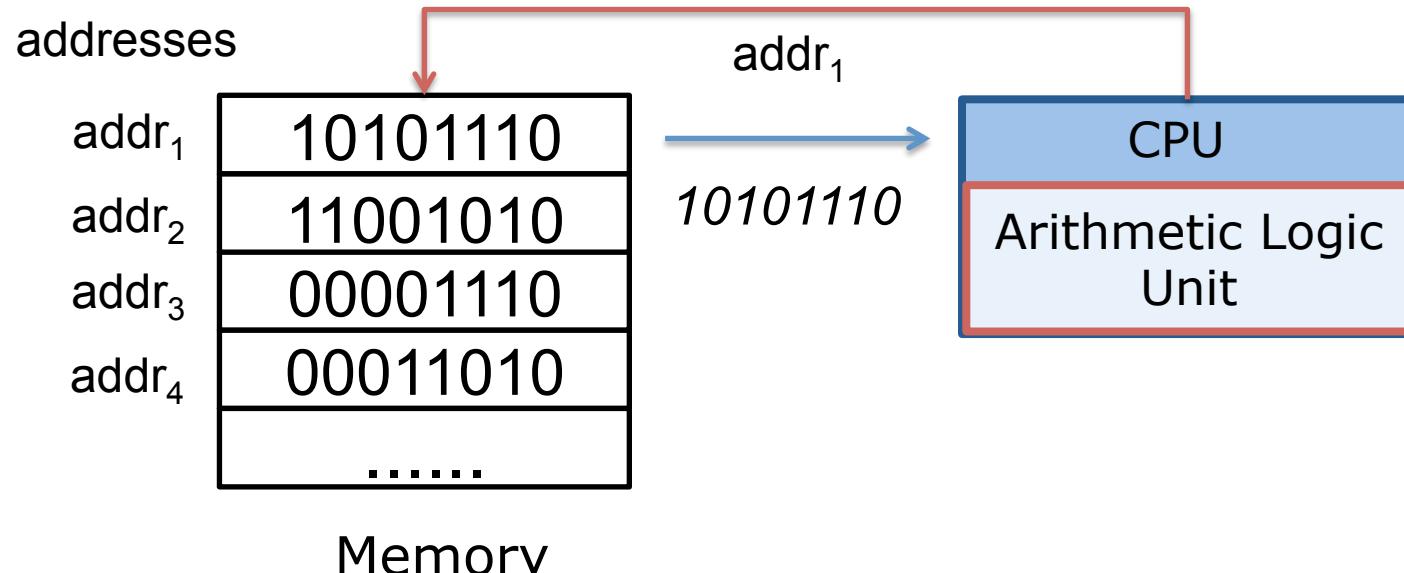


Modern processors

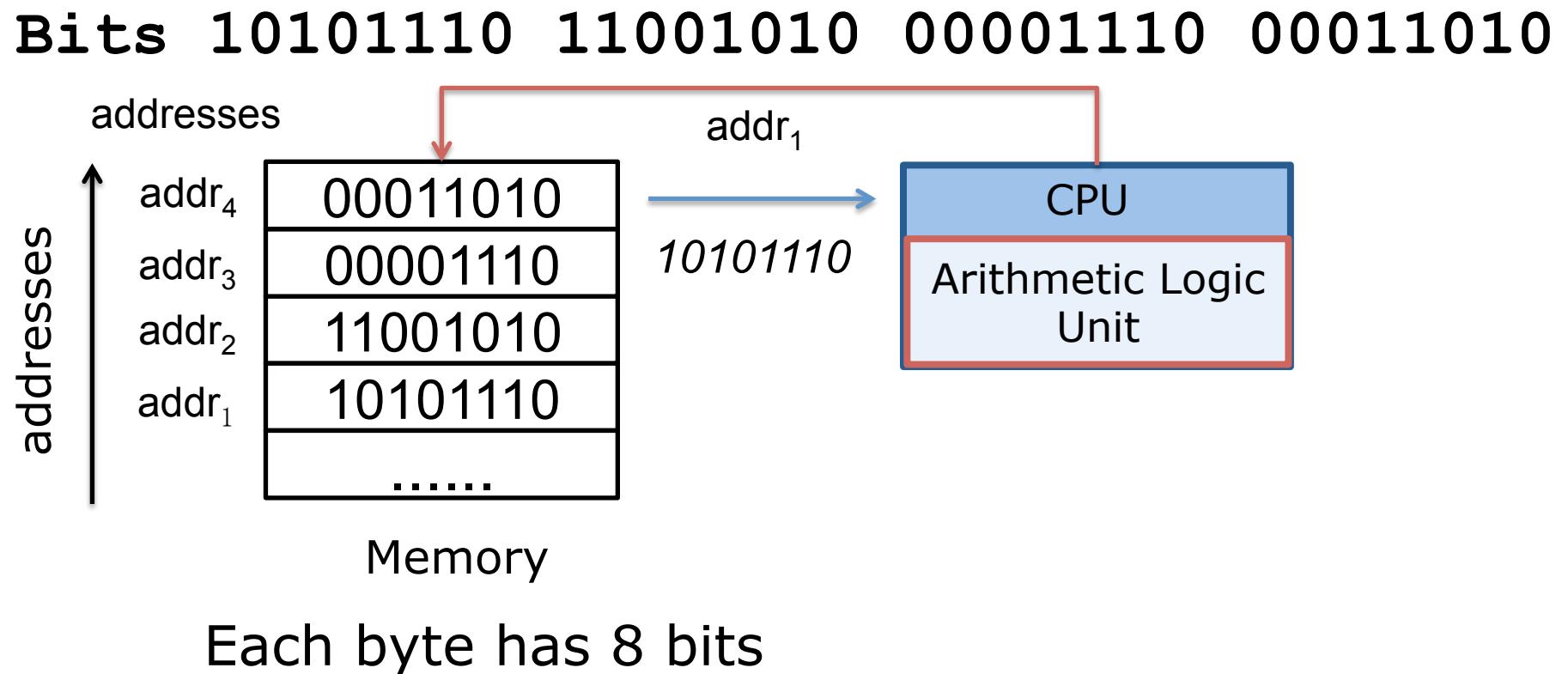
Standard

Your mental model

Bits 10101110 11001010 00001110 00011010



Your mental model



Range of Single Byte

Maximum

Minimum

Range of Single Byte

Maximum

– $1111111_2 \rightarrow 255$

Minimum

Range of Single Byte

Maximum

- $1111111_2 \rightarrow 255$

Minimum

- $0000000_2 \rightarrow 0$

Bit pattern description – intuitive way

Binary notation

Bits 10101110 11001010 00001110 00011010,
4 bytes

Bit pattern description – intuitive way

Binary notation

- Too verbose

Bits 10101110 11001010 00001110 00011010
4 bytes

15 cm on my laptop

Bit pattern description – strawman

Decimal Notation

- how many decimal digits to represent one byte? 3

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26

Bit pattern description – strawman

Decimal Notation

- 3 decimal digits to represent one byte

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	014	026

too tedious to do the conversion

Hexadecimal Notation

Write bit patterns as base-16 (hex) numbers

- Hex “digit” is one of 16 symbols: 0-9, a,b,c,d,e,f
- Each byte is two 4-bit chunks
- Each 4-bit-chunk is represented by a hex “digit”

Hexadecimal “digit”

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex				
Hex (C)				

1010 =

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex	A			
Hex (C)				

$$1010 = 1 * 2^3 + 0 * 2^2 + 1 * 2 + 0 = 10 = A_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	1010	1110	11001010	00001110	00011010
Decimal	174		202		014
Hex		A			026
Hex (C)					

1110 =

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	1010	1110	11001010	00001110	00011010
Decimal	174		202		014
Hex	A	E			026
Hex (C)					

$$1110 = 1 * 2^3 + 1 * 2^2 + 1 * 2 + 0 = 14 = E_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex	A	E		
Hex (C)				

1100 =

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex	A	E	C	
Hex (C)				

$$1100 = 1 * 2^3 + 1 * 2^2 + 0 * 2 + 0 = 12 = C_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	1100	1010	00001110	00011010
Decimal	174		202		014
Hex	A	E	C		026
Hex (C)					

1010 =

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	1100 1010	00001110	00011010
Decimal	174	202	14	26
Hex	A	E	C	A
Hex (C)				

$$1010 = 1 * 2^3 + 0 * 2^2 + 1 * 2 + 0 = 10 = A_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex	A	E	C	A
Hex (C)				0

$$0000 = 0 * 2^3 + 0 * 2^2 + 0 * 2 + 0 = 0 = 0_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	0000 1110	00011010
Decimal	174	202	014	026
Hex	A	E	C	A
Hex (C)			0	E

$$1110 = 1 * 2^3 + 1 * 2^2 + 1 * 2 + 0 = 14 = E_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex	A	E	C	A
Hex (C)				1

$$0001 = 0 * 2^3 + 0 * 2^2 + 0 * 2 + 1 = 1 = 1_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	0001	1010
Decimal	174	202	14	26	
Hex	A	E	C	A	
Hex (C)				0	E

$$1010 = 1 * 2^3 + 0 * 2^2 + 1 * 2 + 0 = 10 = A_{16}$$

Hexadecimal Notation

- Each byte is represented with 2 hex numbers (00_{16} -- FF_{16})

Bits	10101110	11001010	00001110	00011010
Decimal	174	202	14	26
Hex	A	E	C	A
Hex (C)	0xAECA0E1A			

Exercises Time

Hexadecimal

0xBC

Decimal

55

Binary

1010 0111
0011 1110

0xF3

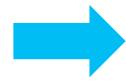
Answers

Hexadecimal	Decimal	Binary
0xA7	$10*16 + 7 = 167$	1010 0111
0x3E	$3*16 + 14 = 62$	0011 1110
0xBC	$11*16 + 12 = 188$	1011 1100
0x37	$3*16+7=55$	0011 0111
0xF3	$15*16+3=243$	1111 0011

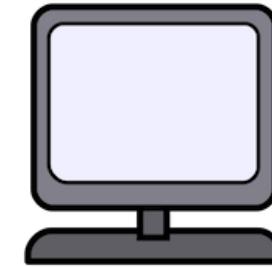
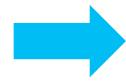
Unsigned addition



$11 + 10$



$0xB + 0xA$



$00001011_2 + 00001010_2$

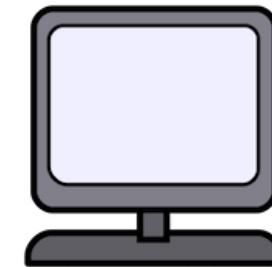
Unsigned addition



$11 + 10$



$0xB + 0xA$



$00001011_2 + 00001010_2$

0 0 0 0 1 0 1 1

+ 0 0 0 0 1 0 1 0

Unsigned addition



$11 + 10$

$0xB + 0xA$

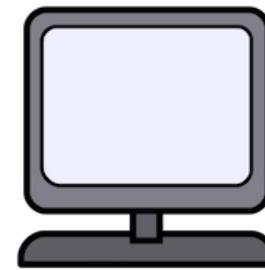
$00001011_2 + 00001010_2$

0 0 0 0 1 0 1 1

+ 0 0 0 0 1 0 1 0

1

Unsigned addition



$11 + 10$

$0xB + 0xA$

$00001011_2 + 00001010_2$

$$\begin{array}{r} 00001011 \\ + 00001010 \\ \hline 21 \end{array}$$

Unsigned addition



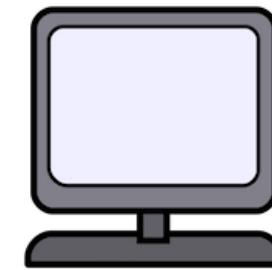
$11 + 10$

$0xB + 0xA$

$00001011_2 + 00001010_2$

$$\begin{array}{r} & & & & & 1 \\ & 0 & 0 & 0 & 0 & 1 & 0 & \textcolor{red}{1} & 1 \\ + & 0 & 0 & 0 & 0 & 1 & 0 & \textcolor{red}{1} & 0 \\ \hline & 0 & 1 & & & & & & \end{array}$$

Unsigned addition



$11 + 10$

$0xB + 0xA$

$00001011_2 + 00001010_2$

$$\begin{array}{r} & & & & & 1 \\ & & & & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ + & & & & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ \hline & & & & & 1 & 0 & 1 \end{array}$$

Unsigned addition



$11 + 10$

$0xB + 0xA$

$00001011_2 + 00001010_2$

$$\begin{array}{r} 00001011 \\ + 00001010 \\ \hline 00010101 \end{array}$$

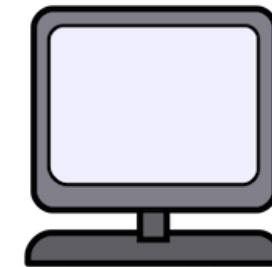
Unsigned subtraction



11 - 10



0xB - 0xA



00001011₂ - 00001010₂

$$\begin{array}{r} 00001011 \\ - 00001010 \\ \hline 00000001 \end{array}$$

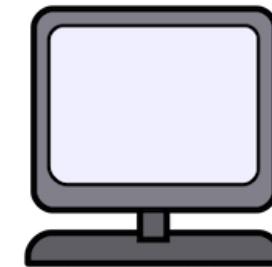
Unsigned subtraction



11 - 10



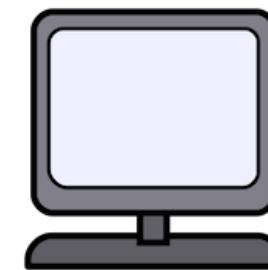
0xB - 0xA



00001011₂ - 00001010₂

$$\begin{array}{r} 00001011 \\ - 00001010 \\ \hline 00000001 \end{array}$$

Unsigned subtraction



10 - 11

0xA - 0xB

00001010₂ - 00001011₂

$$\begin{array}{r} 00001010 \\ - 00001011 \\ \hline \end{array}$$

? ? ?

Question:
How to represent negative numbers?

Strawman

Most significant bit (MSB) represent the sign

$$0\ 0\ 0\ 0\ 0\ 0\ 1_2 \rightarrow 1$$

$$1\ 0\ 0\ 0\ 0\ 0\ 1_2 \rightarrow -1$$

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 0\ 1 \\ +\ 1\ 0\ 0\ 0\ 0\ 0\ 1 \\ \hline \end{array}$$

Strawman

Most significant bit (MSB) represent the sign

$$0\ 0\ 0\ 0\ 0\ 0\ 1_2 \rightarrow 1$$

$$1\ 0\ 0\ 0\ 0\ 0\ 1_2 \rightarrow -1$$

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 0\ 1 \\ +\ 1\ 0\ 0\ 0\ 0\ 0\ 1 \\ \hline 1\ 0\ 0\ 0\ 0\ 1\ 0 \end{array}$$

Strawman

Most significant bit (MSB) represent the sign

$$0\ 0\ 0\ 0\ 0\ 0\ 1_2 \rightarrow 1$$

$$1\ 0\ 0\ 0\ 0\ 0\ 1_2 \rightarrow -1$$

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 0\ 1 \\ +\ 1\ 0\ 0\ 0\ 0\ 0\ 1 \\ \hline \end{array}$$

$$1\ 0\ 0\ 0\ 0\ 1\ 0$$

-2 ???



Two's complement

Byte 10010110

Unsigned number

$$1 * 2^7 + 0 * 2^6 + 0 * 2^5 + 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 1 * 2 + 0 * 2^0$$

Two's complement

Byte 10010110

Unsigned number

$$1 * 2^7 + 0 * 2^6 + 0 * 2^5 + 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 1 * 2 + 0 * 2^0$$

Signed number

$$-1 * 2^7 + 0 * 2^6 + 0 * 2^5 + 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 1 * 2 + 0 * 2^0$$

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0] \quad val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

MSB: $val(b_w) = -b_w * 2^w$

Other: $val(b_i) = b_i * 2^i, \quad 0 \leq i < w$

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0]$$

$$val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

Binary

1000 0001

1010 0101

0101 0101

Value

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0] \quad val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

Binary	Value
1000 0001	-1 * $2^7 + 1$
1010 0101	
0101 0101	

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0] \quad val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

Binary		Value
1000 0001	$-1 * 2^7 + 1$	-127
1010 0101	$-1 * 2^7 + 2^5 + 2^2 + 2^0$	-91
0101 0101		

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0]$$

$$val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

Binary		Value
1000 0001	$-1 * 2^7 + 1$	-127
1010 0101	$-1 * 2^7 + 2^5 + 2^2 + 2^0$	-91
0101 0101	$2^6 + 2^4 + 2^2 + 2$	85

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0]$$

$$val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

$$\begin{array}{r} 00000001 \\ + 10000001 \\ \hline 1000010 \end{array}$$

Two's complement

$$\vec{b} = [b_w, b_{w-1}, \dots, b_0]$$

$$val(\vec{b}) = -b_w 2^w + \sum_{i=0}^{w-1} b_i 2^i$$

$$\begin{array}{r} 00000001 \\ + 10000001 \\ \hline \end{array}$$

$$\begin{array}{r} 2^0 & 1 \\ -1 * 2^7 + 2^0 & -127 \\ \hline \end{array}$$

$$10000010$$

$$\begin{array}{r} -1 * 2^7 + 2^1 \\ -126 \end{array}$$

Find 2's complement quickly

With a negative number, how to give its binary representation? e.g. -40

Find 2's complement quickly

With a negative number, how to give its binary representation? e.g. -40

Step 1. represent 40 in binary

0010 1000

Find 2's complement quickly

With a negative number, how to give its binary representation? e.g. -40

Step 2. flip all bits

0010 1000 → 1101 0111

Find 2's complement quickly

With a negative number, how to give its binary representation? e.g. -40

Step 3. add 1

0010 1000 → 1101 0111 → 1101 1000

Find 2's complement quickly

With a negative number, how to give its binary representation? e.g. -40

Step 3. add 1

0010 1000 → 1101 0111 → 1101 1000

$$\begin{array}{r} 0010\ 1000 \\ +\ 1101\ 0111 \\ \hline 1111\ 1111 \end{array}$$

Find 2's complement quickly

With a negative number, how to give its binary representation? e.g. -40

Step 3. add 1

0010 1000 → 1101 0111 → 1101 1000

$$\begin{array}{r} 0010\ 1000 \\ 1101\ 0111 \\ + \quad \quad \quad 1 \\ \hline 1\ 0000\ 0000 \end{array}$$

Why does this trick work

- What is $1111\dots11_2$ in 2's complement?

- $\vec{b} + (\sim \vec{b}) = 11\dots11_2 = -1$



b with bits
flipped

$$-\vec{b} = (\sim \vec{b}) + 1$$

Exercise Time II

Hexadecimal

0xce

Decimal

127

-128

-90

Binary

1001 1100

Answers

Hexadecimal	Decimal	Binary
0xce	-50	1100 1110
0x9c	-100	1001 1100
0x7f	127	0111 1111
0x80	-128	1000 0000
0xa6	-90	1010 0110

Ranges

	Range	Min	Max
1 byte unsigned	$[0, 2^8 - 1]$	0	255
1 byte signed			

Ranges

	Range	Min	Max
1 byte unsigned	$[0, 2^8 - 1]$	0	255
1 byte signed	$[-2^7, 2^7 - 1]$	-128	127

Min: 1000 0000

Max: 0111 1111

Overflow

$$\begin{array}{r} 10000001 \\ + 10000001 \\ \hline \end{array} \quad \begin{array}{l} -127 \\ -127 \end{array}$$

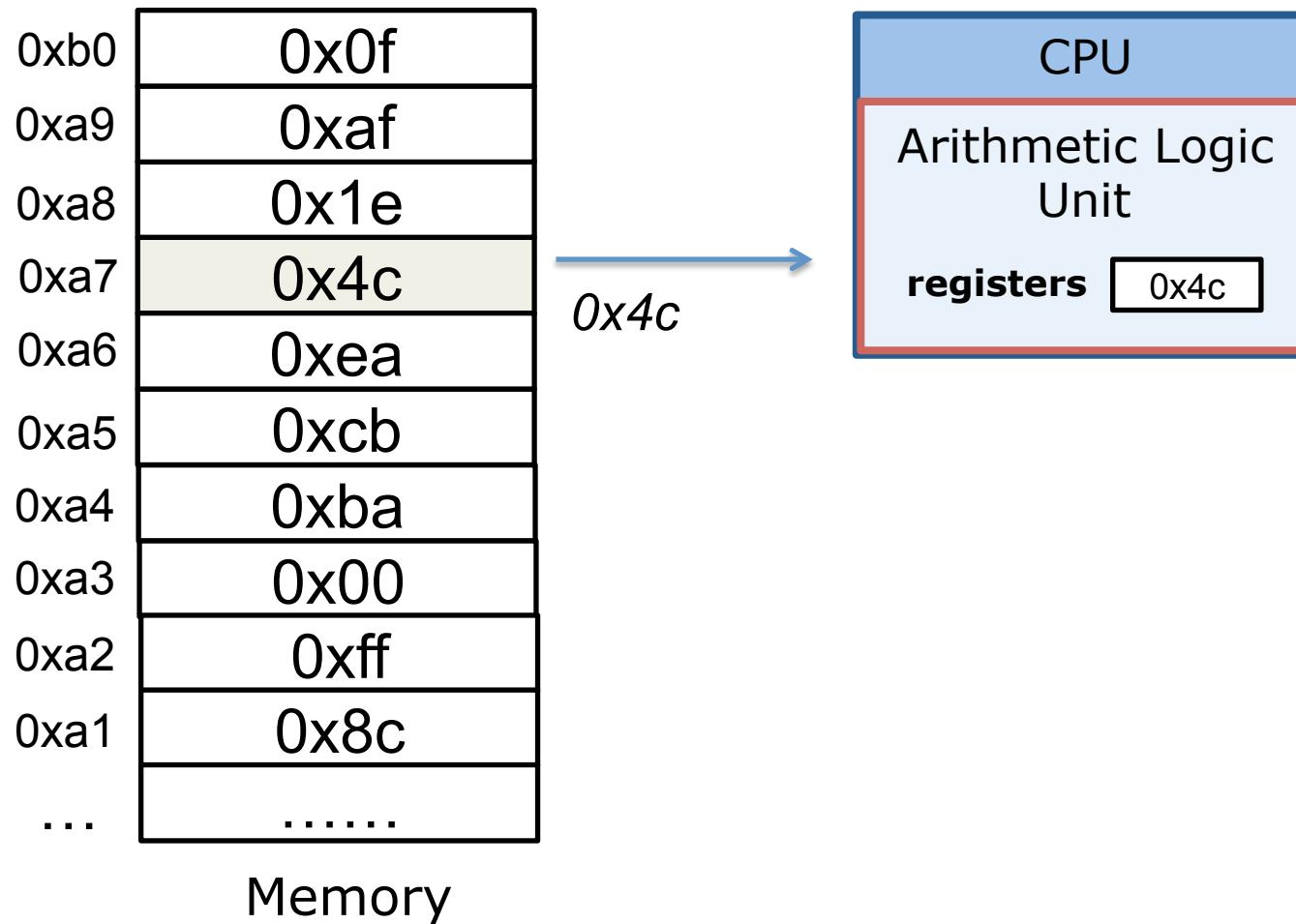
	Range	Min	Max
1 byte unsigned	$[0, 2^8 - 1]$	0	255
1 byte signed	$[-2^7, 2^7 - 1]$	-128	127

Overflow

$$\begin{array}{r} 10000001 \\ + 10000001 \\ \hline 100000010 \end{array} \quad \begin{array}{l} -127 \\ -127 \\ 2 ??? \end{array} \quad \begin{array}{c} \text{困惑表情} \end{array}$$

	Range	Min	Max
1 byte unsigned	$[0, 2^8 - 1]$	0	255
1 byte signed	$[-2^7, 2^7 - 1]$	-128	127

Intel 8080



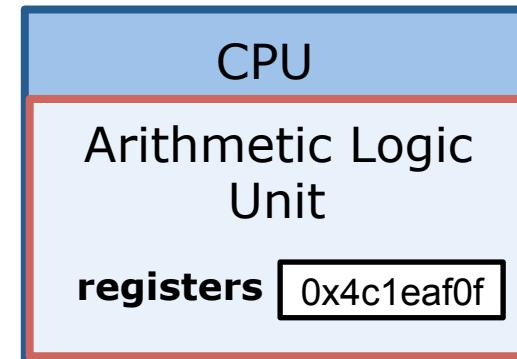
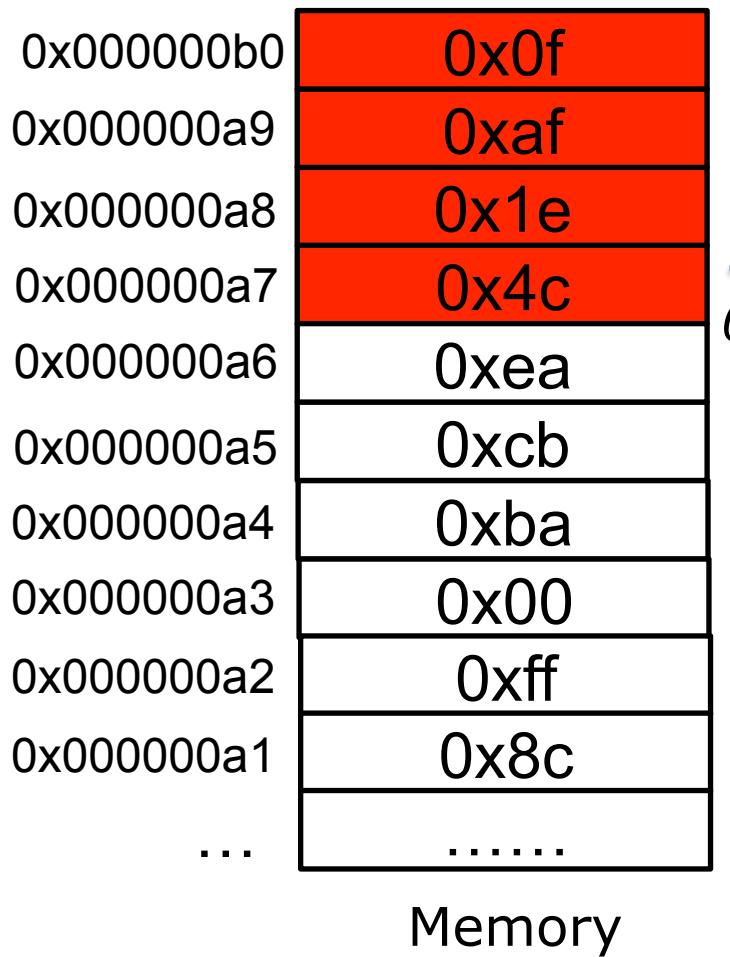
Intel 8080



8 bits machine – 8 bits length of

- Memory – processor transfer
- CPU Register
- Memory Address
- Instruction

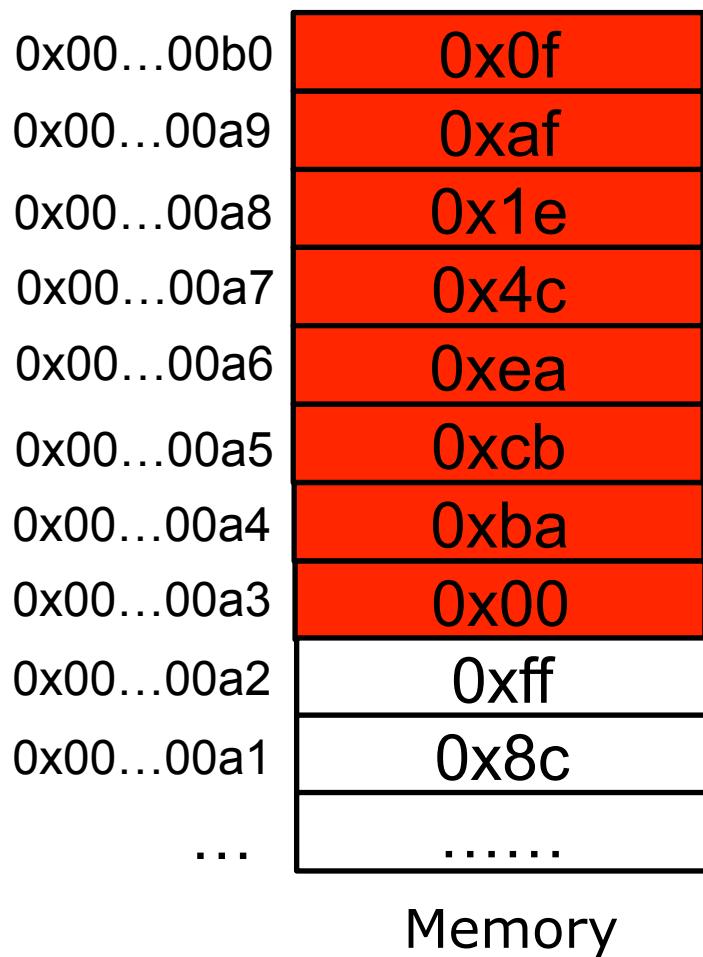
Intel 386



32 bits machine – 32 bits length of

- Memory – processor transfer
- CPU Register
- Memory Address
- Instruction

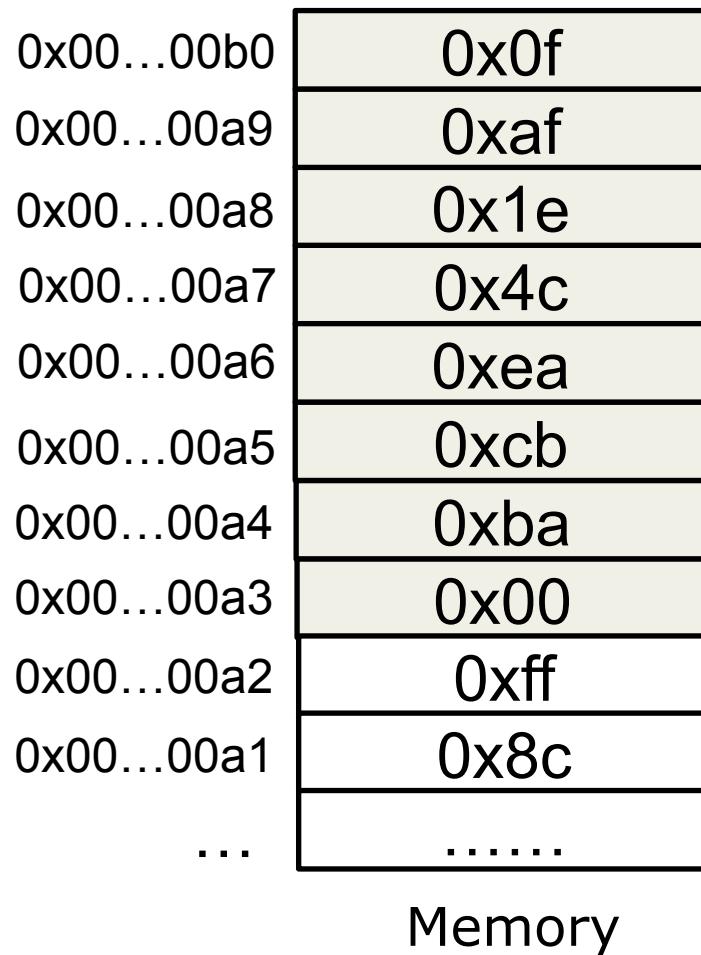
Intel Opteron → i7



64 bits machine – 64 bits length of

- Memory – processor transfer
- CPU Register
- Memory Address
- Instruction

Intel Opteron → i7



Word

- Memory – processor transfer
- CPU Register
- Memory Address
- Instruction

Word

Definition

- Fixed size of data handled as a unit by the instruction set or processor

Length

- 8 for 8 bits machine
- 32 for 32 bits machine
- 64 for 64 bits machine

C's integral data types on 64 bits machine

	Length	Min	Max
[signed] char	1 byte	-2^7	$2^7 - 1$
unsigned char	1 byte	0	$2^8 - 1$
short	2 bytes		
unsigned short	2 bytes		
int	4 bytes		
unsigned int	4 bytes		
long	8 bytes		
unsigned long	8 bytes		

Integral data types on 64 bits machine

	Length	Min	Max
[signed] char	1 byte	-2^7	$2^7 - 1$
unsigned char	1 byte	0	$2^8 - 1$
short	2 bytes	-2^{15}	$2^{15} - 1$
unsigned short	2 bytes	0	$2^{16} - 1$
int	4 bytes		
unsigned int	4 bytes		
long	8 bytes		
unsigned long	8 bytes		

Integral data types on 64 bits machine

	Length	Min	Max
[signed] char	1 byte	-2^7	$2^7 - 1$
unsigned char	1 byte	0	$2^8 - 1$
short	2 bytes	-2^{15}	$2^{15} - 1$
unsigned short	2 bytes	0	$2^{16} - 1$
int	4 bytes	-2^{31}	$2^{31} - 1$
unsigned int	4 bytes	0	$2^{32} - 1$
long	8 bytes		
unsigned long	8 bytes		

Integral data types on 64 bits machine

	Length	Min	Max
[signed] char	1 byte	-2^7	$2^7 - 1$
unsigned char	1 byte	0	$2^8 - 1$
short	2 bytes	-2^{15}	$2^{15} - 1$
unsigned short	2 bytes	0	$2^{16} - 1$
int	4 bytes	-2^{31}	$2^{31} - 1$
unsigned int	4 bytes	0	$2^{32} - 1$
long	8 bytes	-2^{63}	$2^{63} - 1$
unsigned long	8 bytes	0	$2^{64} - 1$

Your first C program

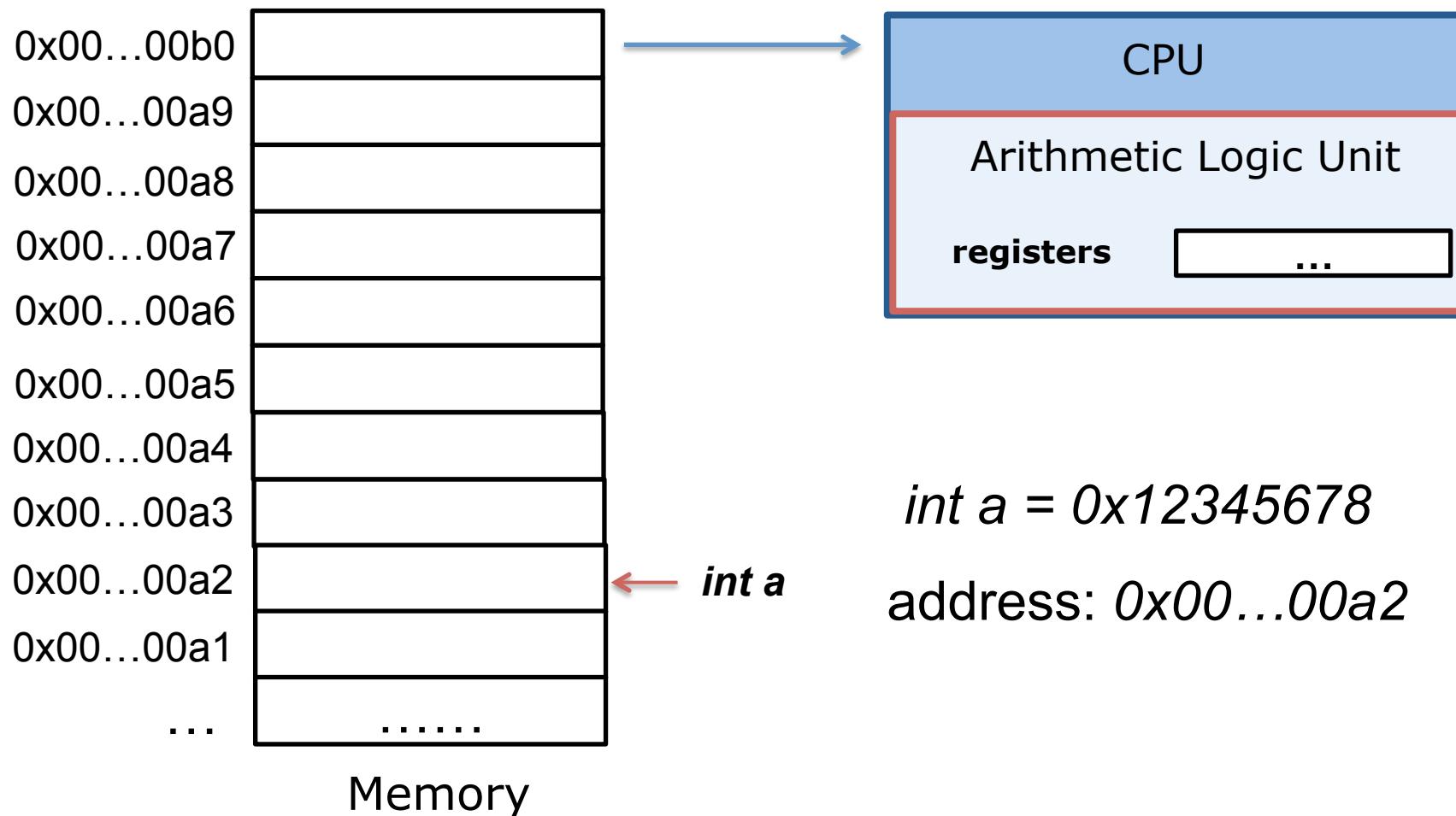
```
#include <stdio.h>

int
main()
{
    char x = -127;
    char y = 0x81;
    char z = x + y;
    printf("hello world sum is %d\n", z);
}
```

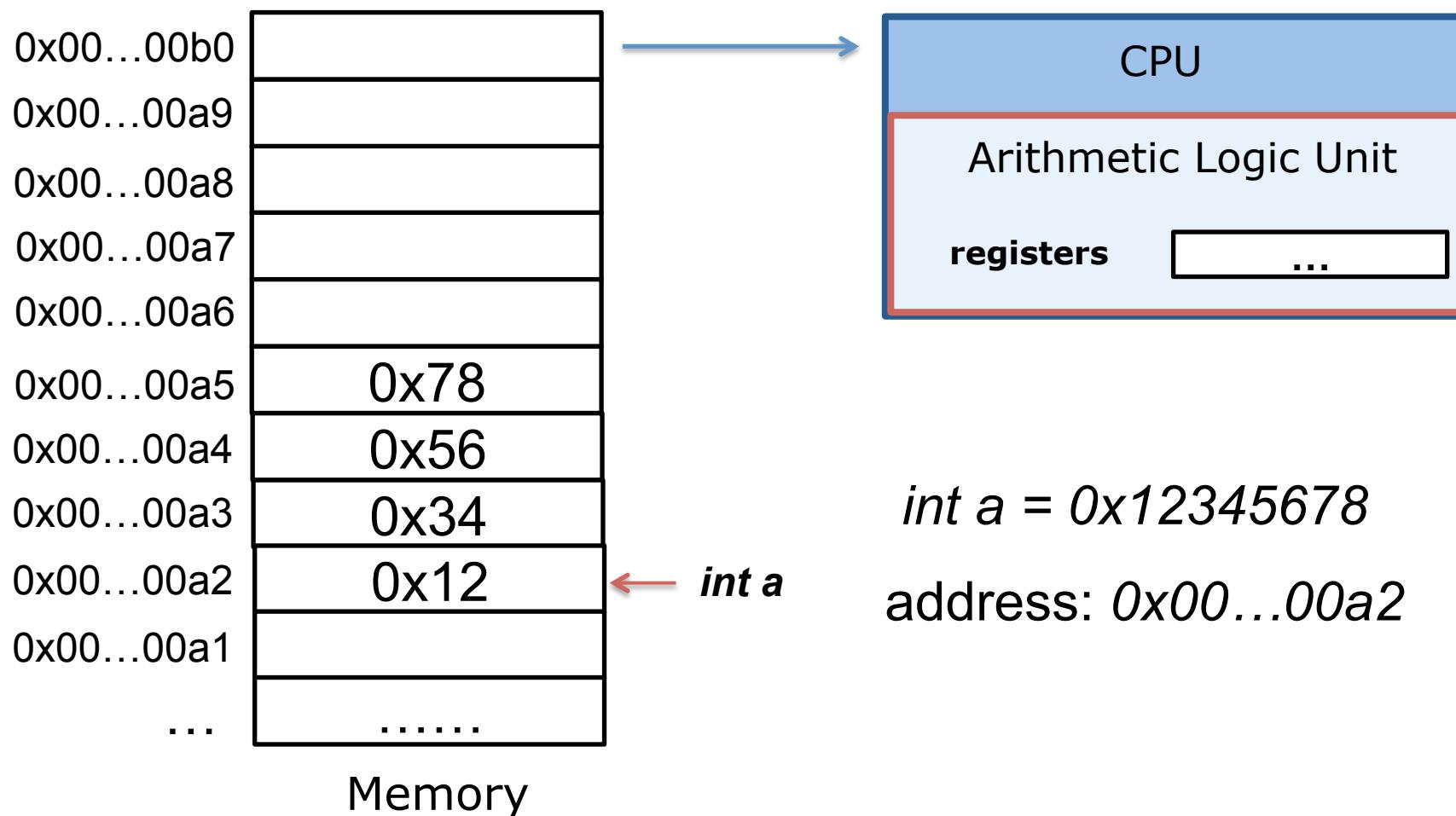
```
$ gcc helloworld.c
$ ./a.out
```

$$\begin{array}{r} 10000001 \\ + 10000001 \\ \hline 10000010 \end{array} \quad \begin{array}{l} -127 \\ -127 \\ 2 \end{array}$$

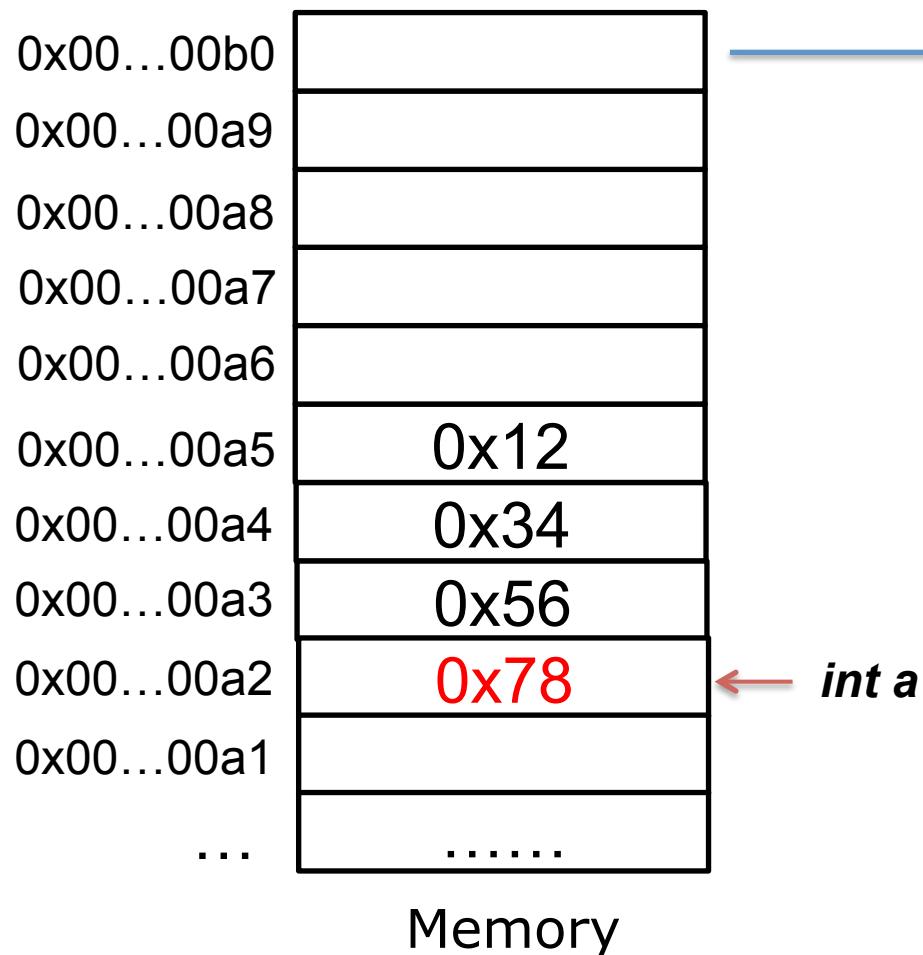
Memory layout



Memory layout – Intuition



Memory layout – LittleEndian



int a = 0x12345678

address: 0x00...00a2

Least significant byte in
smallest address,
e.g., laptops, and server machines

Advantages of Little Endian

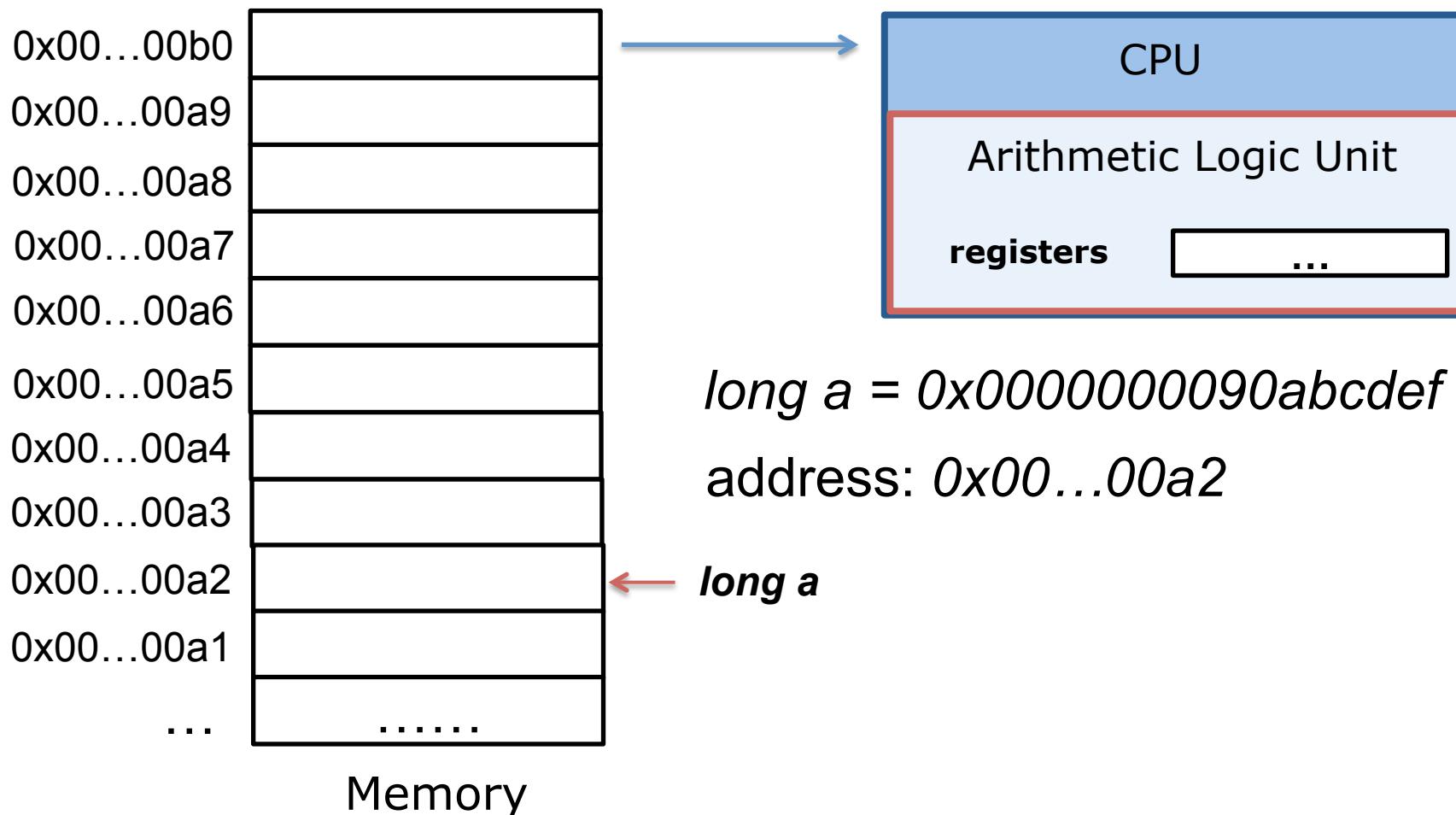
$$\begin{array}{r} 0x12345678 \\ + 0x12131415 \\ \hline \end{array}$$


Processor performs the calculation from the least significant bit

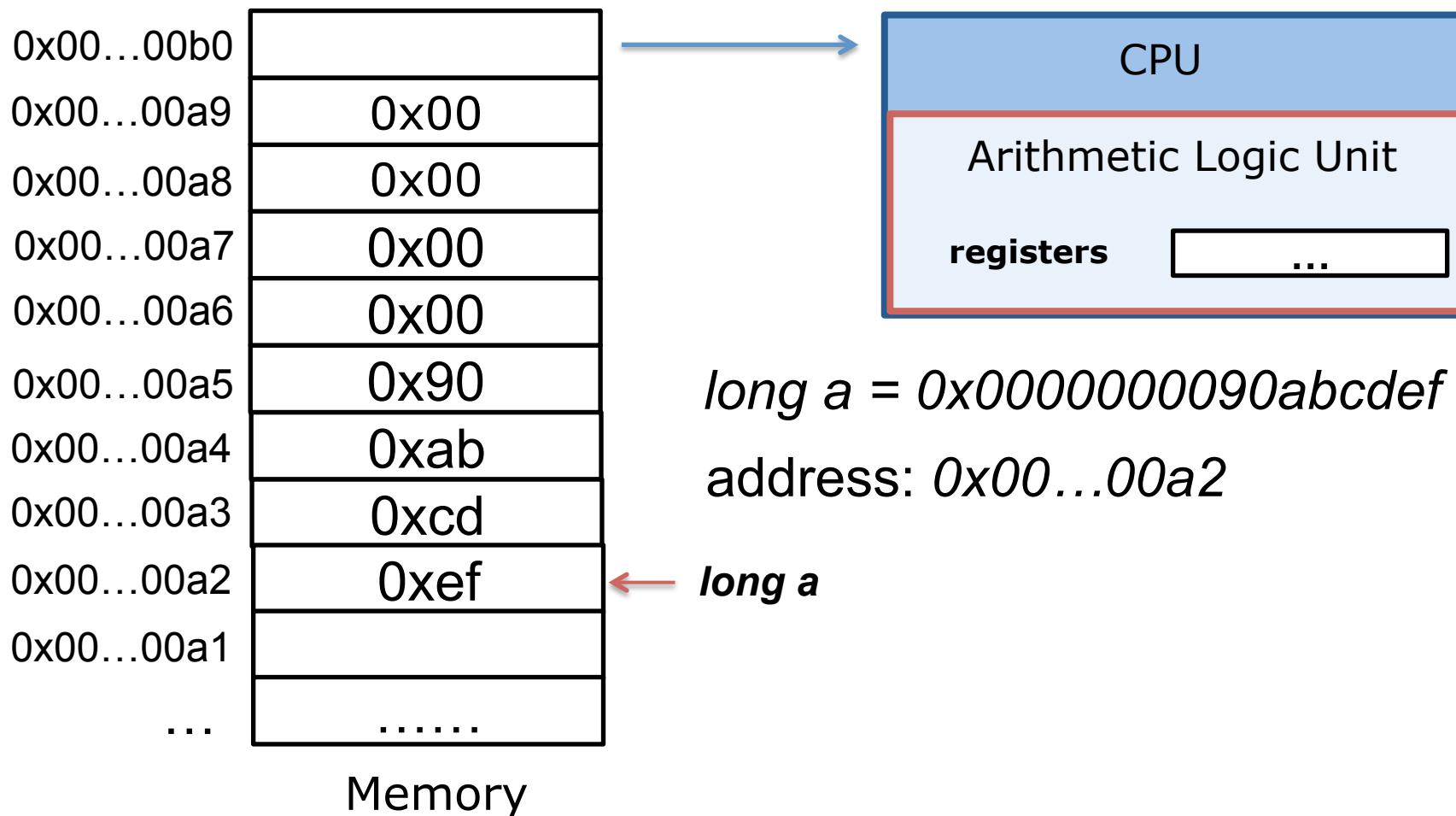


Processor can simultaneously perform memory transfer and calculation.

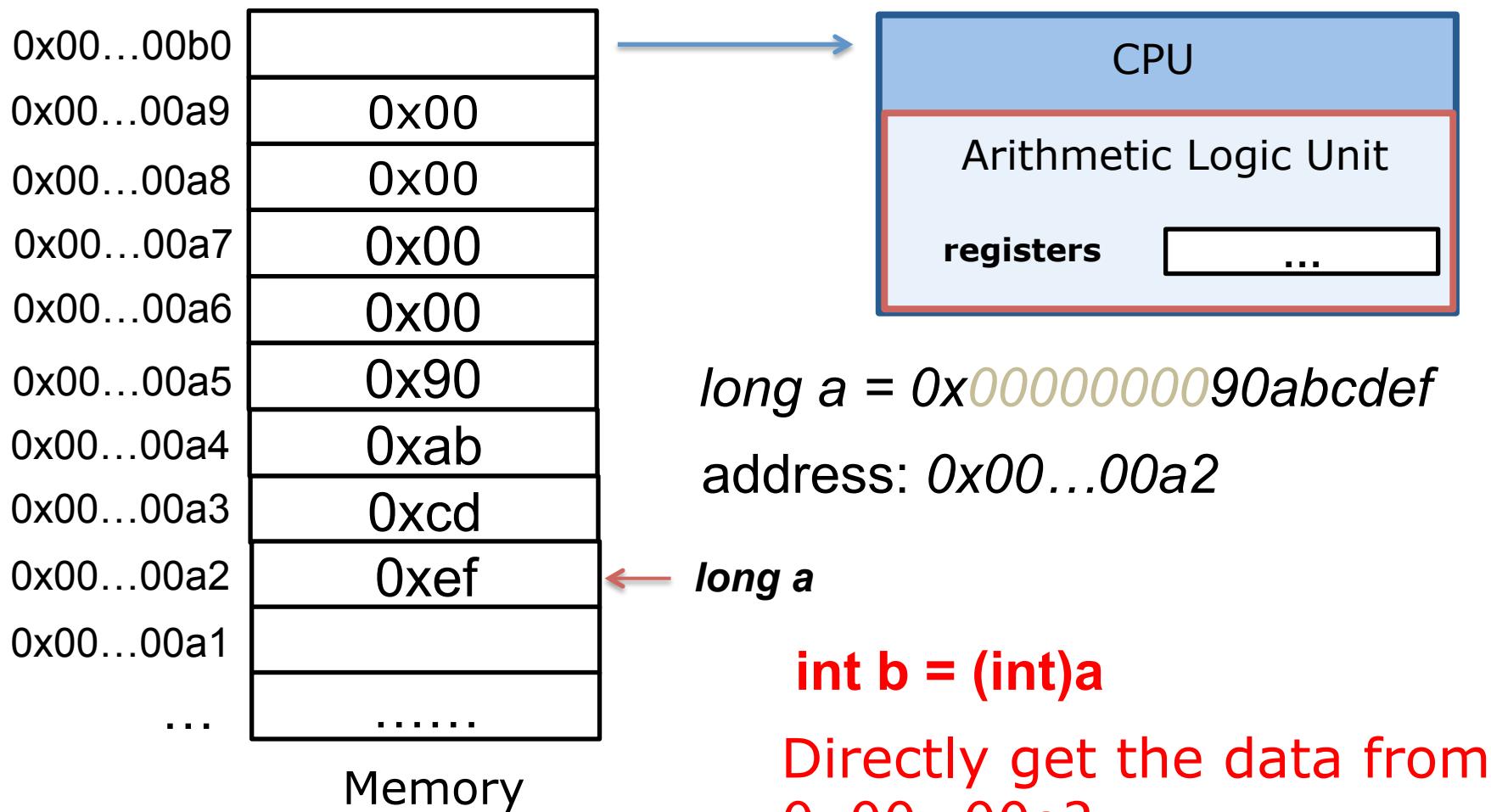
Advantages of Little Endian



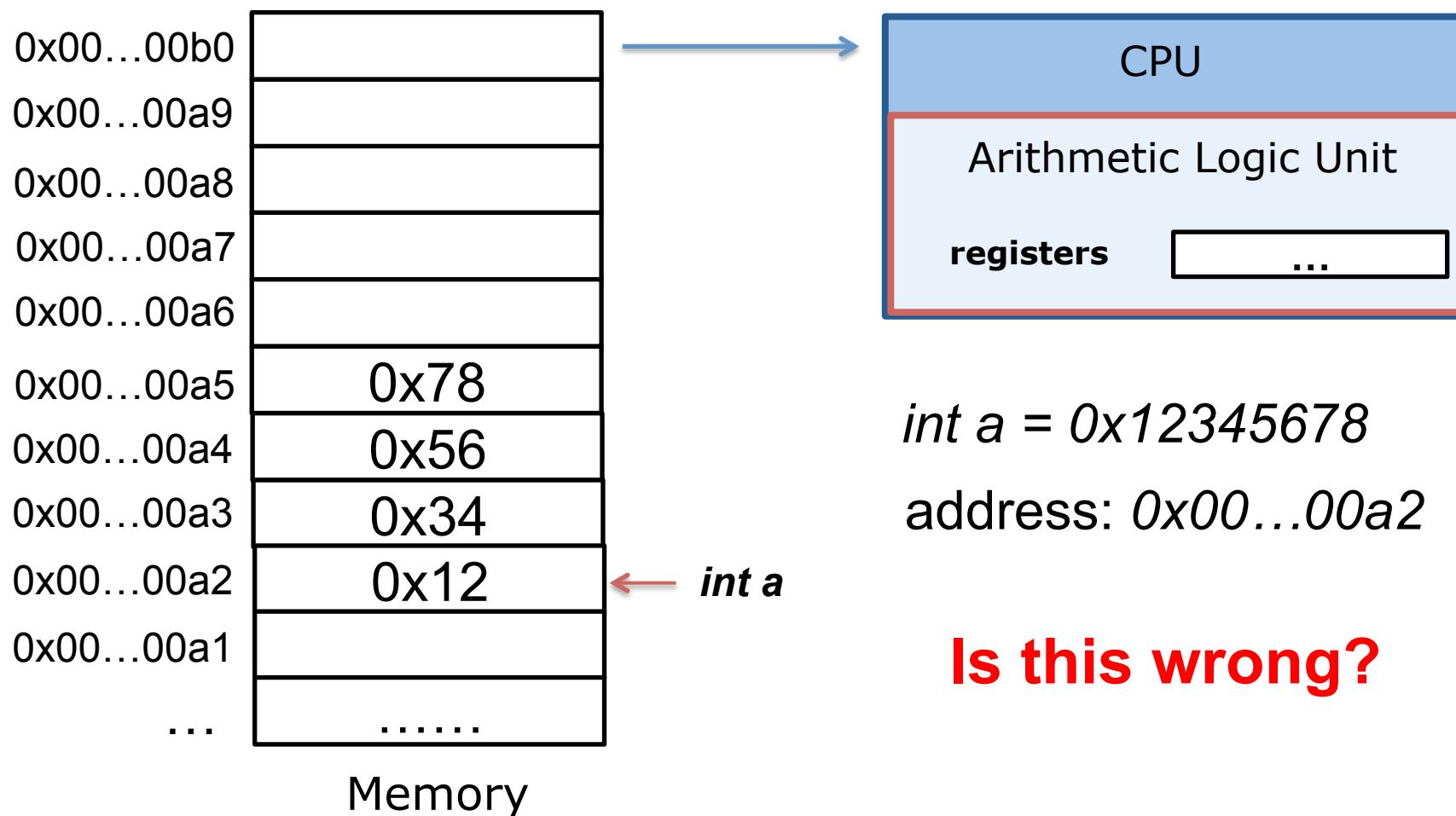
Advantages of Little Endian



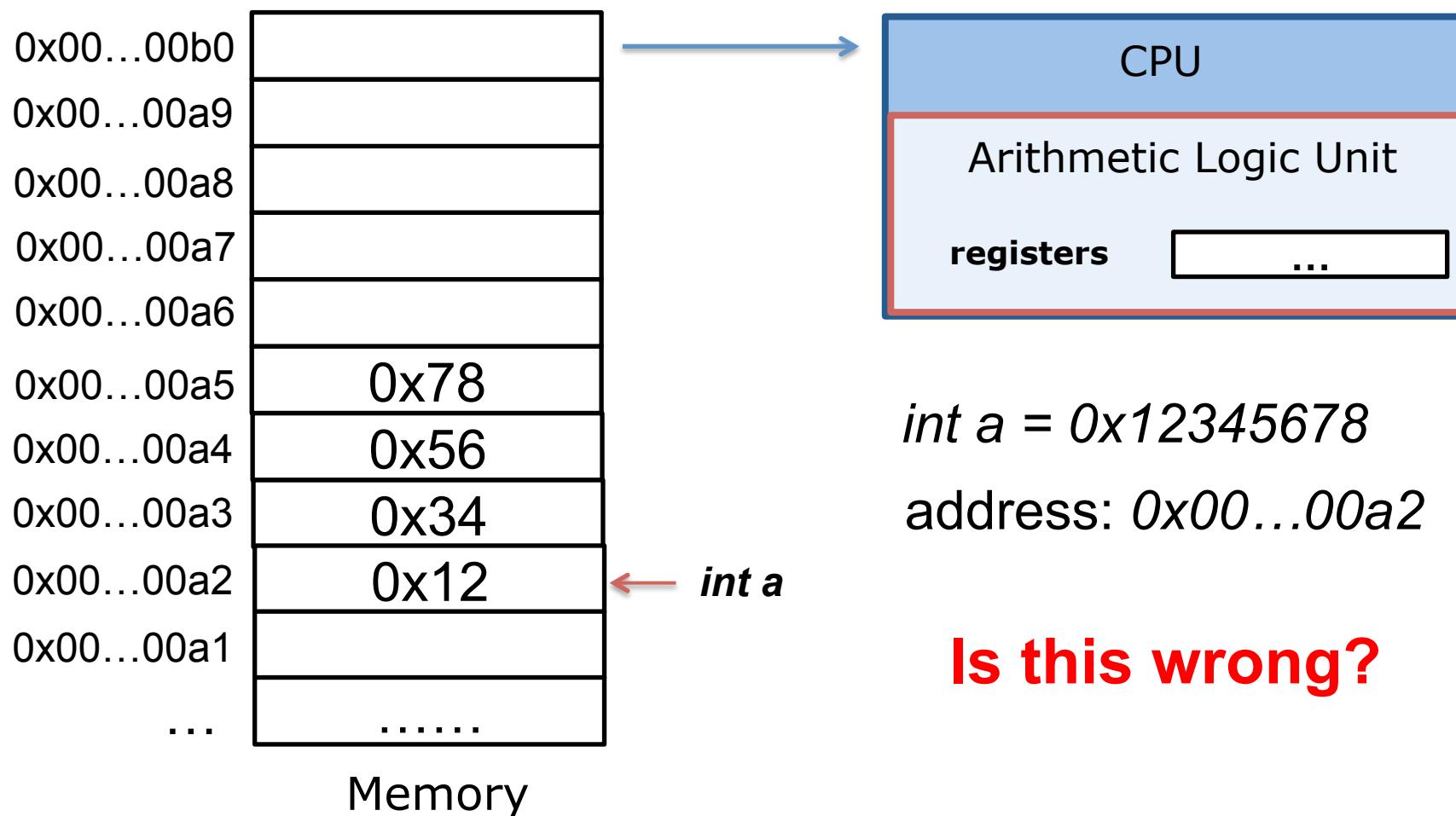
Advantages of Little Endian



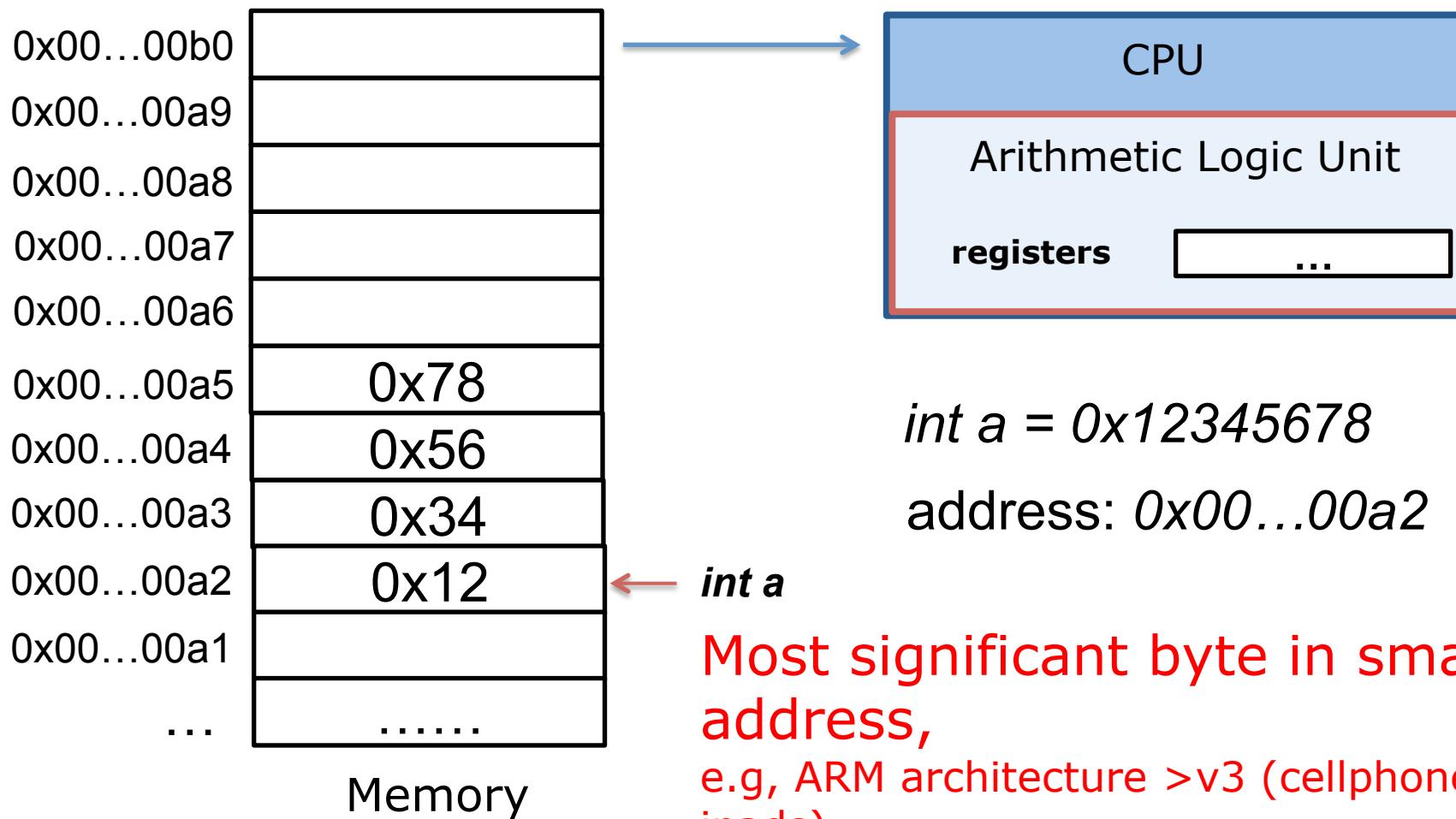
Memory layout – Intuition



Memory layout – Intuition



Memory layout – Big Endian



Advantages of Big Endian

1. Easy to read
2. Test whether the number is positive or negative by looking at the byte at offset zero.